

D-brane width

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Abstract

Loop quantum gravity predicts that there are non-zero minimal area, and non-zero minimal volume in (3+1) dimensions. Given this, one can easily guess that one will have a non-zero minimal 9-volume in (9+1) dimensions. Therefore, in this paper, we argue that not only D9-brane but also Dp-brane for p less than 9 has a 9-volume. This idea is new, as the present view states that such a Dp-brane has p-volume but no 9 volume. To demonstrate this, first, we equate D8-brane action with D9-brane action and show that 9th direction which is perpendicular to D8-brane has non-zero width. We repeat this step for different ps; we equate Dp-brane action with D_{p-1} brane action. By this iteration and induction we conclude that Dp-brane has non-zero widths for each of (9-p) directions perpendicular to the Dp-brane, and therefore, non-zero 9 volume. When antisymmetric tensor and field strength are zero, this width is calculated to be $2\pi\sqrt{\alpha'}$ for all (9-p) directions. For non-vanishing antisymmetric tensor and field strength, the width receives small corrections. In this paper, we only calculate up to the first order correction.

The discovery of D-brane in mid 90s opened a new world for string theorists. [1] However, it was not known that all Dp-branes, regardless of p, have 9-volume, and therefore, non-zero widths, for p less than 9. This must be true, because one may easily guess that there is a non-zero minimal 9-volume in (9+1) dimensions, as there are non-zero minimal area and non-zero minimal volume in (3+1) dimensions according to loop quantum gravity. [2] Therefore, in this paper, we will argue that Dp-branes have 9-volume, and therefore have non-zero widths for p less than 9. This is in contrast with the present view that Dp-branes have zero 9 volume if p is less than 9. For example, according to the present view, a D3-brane may have 3-volume, but no 9-volume, because 6=9-3 spatial directions perpendicular to D3-brane is infinitely thin, or equal to zero length. However, as I will show Dp-brane can have widths for (9-p) directions perpendicular to the brane, it can have non-zero 9-volume.

To demonstrate this, we will first argue that the formula for D9-brane action can be used for any Dp-brane for arbitrary p, when suitably interpreted. This is justified, if one assumes that Dp-brane and D9-brane are made out of the same "matter," therefore, if D9-brane tension is used as universal tension for any p. This means that for vanishing antisymmetric tensor and field strength as D9-brane action is D9-brane tension multiplied by ten-dimensional world volume swiped by D9-brane, Dp-brane action is D9-brane tension multiplied by ten-dimensional world volume swiped by Dp-brane. To understand this explicitly, let's closely look at Dp-brane action which can be found in [3]. For the Dp-brane, the action is following.

$$S = T_p e^{-\phi} \int d^{p+1}\xi \sqrt{-\det(G_{ab} + B_{ab} + 2\pi\alpha' F_{ab})} \quad (1)$$

For p=9, we

$$S = T_9 e^{-\phi} \int d^{10}\xi \sqrt{-\det(G_{ab} + B_{ab} + 2\pi\alpha' F_{ab})} \quad (2)$$

where a and b run from 0 to 9. For p=8 we get

$$S = T_8 e^{-\phi} \int d^9\xi \sqrt{-\det(G_{\mu\nu} + B_{\mu\nu} + 2\pi\alpha' F_{\mu\nu})} \quad (3)$$

where μ and ν run from 0 to 8. Now, let's use formula (2), a D9-brane action formula, to calculate D8-brane action, as claimed. First, note that

$$d^{10}x_i = d^9x_i dx_9 \quad (4)$$

Here, x_9 is the direction perpendicular to D8-brane. Plugging this relation to (2), we get

$$\begin{aligned} S &= T_9 e^{-\phi} \int d^9x_i dx_9 \sqrt{-\det(G_{ab} + B_{ab} + 2\pi\alpha' F_{ab})} \\ &= T_8 e^{-\phi} \int d^9x_i \sqrt{-\det(G_{\mu\nu} + B_{\mu\nu} + 2\pi\alpha' F_{\mu\nu})} \end{aligned} \quad (5)$$

To simplify this equation, note that,

$$\begin{aligned}
& \sqrt{-\det(G_{ab} + B_{ab} + 2\pi\alpha' F_{ab})} \\
&= \sqrt{-G_{99} \det(G_{\mu\nu} + B_{\mu\nu} + 2\pi\alpha' F_{\mu\nu}) - \sum_{\mu=1}^9 \frac{(B_{9\mu} + 2\pi\alpha' F_{9\mu})(B_{\mu 9} + 2\pi\alpha' F_{\mu 9})(\det G_{ab})}{G_{\mu\mu} G_{99}}}
\end{aligned} \tag{6}$$

Here, we only considered up to the first order of antisymmetric tensor and field strength and assumed G_{ab} 's are diagonalized. Given this, by another one more approximation, we get the following.

$$\begin{aligned}
& \frac{\sqrt{-\det(G_{ab} + B_{ab} + 2\pi\alpha' F_{ab})}}{\sqrt{-\det(G_{\mu\nu} + B_{\mu\nu} + 2\pi\alpha' F_{\mu\nu})}} \\
&= \sqrt{G_{99} - \sum_{\mu=1}^9 \frac{(B_{9\mu} + 2\pi\alpha' F_{9\mu})(B_{\mu 9} + 2\pi\alpha' F_{\mu 9})}{G_{\mu\mu}}}
\end{aligned} \tag{7}$$

Also note the following.

$$\int \sqrt{G_{99}} dx_9 = w_9 \tag{8}$$

where w_9 is the width of D8-brane, as $\sqrt{G_{99}}$ is the correct Jacobian. One more thing that we have to consider is the relationship between T_9 and T_8 . A simple formula can be found in [3]. That is

$$T_p = \frac{T_{p-1}}{2\pi\sqrt{\alpha'}} \tag{9}$$

When $p=9$, we get

$$T_9 = \frac{T_8}{2\pi\sqrt{\alpha'}}$$

Plugging (7),(8),(9) into (5) we get

$$\begin{aligned}
w_9 &= 2\pi \frac{\sqrt{\alpha'}}{\sqrt{1 + (B_{\mu 9} + 2\pi\alpha' F_{\mu 9})^2 + O(F^4)}} \\
&= 2\pi\sqrt{\alpha'} \left(1 - \frac{1}{2}(B_{\mu 9} + 2\pi\alpha' F_{\mu 9})^2 + O(F^4) \right)
\end{aligned} \tag{10}$$

Therefore, we got a formula for D8-brane width in the direction perpendicular to D8-brane. This whole step ((2) to (10)) can be done iteratively, for other values of p . For example, comparing D8-brane action and D7-brane action will give the width of D7-brane in the 8th direction, as comparing D9-brane action and D8-brane action gave the width of D8 brane in the 9th direction. Therefore, we arrive at the conclusion that Dp-brane has widths for $(9-p)$ directions perpendicular to the Dp-brane. This width was calculated to be $2\pi\sqrt{\alpha'}$ for vanishing antisymmetric tensor and field strength and receives a small correction when they are non-vanishing. Also, notice that D-brane width is on the order of string scale.

In conclusion, D-branes cannot be infinitely thin; they have non-zero widths. Even D0-brane has non-zero widths, therefore it has a non-zero finite size or a 9-volume.

References

- [1] J. Polchinski "Dirichlet-Branes and Ramond-Ramond Charges," Phys. Rev. Lett. 75 (1995) 4724 [hep-th/9510017]
- [2] Carlo Rovelli and Lee Smolin "Discreteness of area and volume in quantum gravity," Nucl. Phys. B 442, 593 (1995) [Erratum-ibid. B 456, 753 (1995)] [arXiv:gr-qc/9411005].
- [3] Joseph Polchinski "String Theory, Vol 1"